

SSAOS 2025

Blansko, Czech Republic

7-12 September



**Faculty of Mathematics
and Physics**
Charles University



Faculty of
Engineering

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Schedule

Monday Morning

9:00	Paolo Aglianò Ideals in universal algebra invited
10:00	Reinhard Pöschel How generalized quasiorders appear in rectangular algebras
10:20	Andreja Tepavčević Role of weak congruences in theoretical and practical applications
10:40	Coffee
11:10	Paolo Marimon Minimal operations over permutation groups invited
12:10	Johanna Brunar A lope towards loops: temporal digraphs
12:30	Maximilian Hadek $Kőnig = Ramsey$
	Lunch

Monday Evening

	Dinner
19:30	Moritz Albert Schöbi Algebraicity is irrelevant (in the dichotomy conjecture for infinite-domain constraint satisfaction problems)
19:50	Christoph Spiess An ω -categorical cheese platter for finite-domain Promise Constraint Satisfaction Problems
20:10	Radka Schwartzová Clones between Group $(\mathbb{Z}_8, +)$ and Ring $(\mathbb{Z}_8, +, \cdot)$
20:30	Short break
20:40	Juraj Hirjak, Lucia Kőszegyová Meet-irreducibility of congruence lattices of prime-cycled algebras
21:00	Michael Kompatscher Mal'cev algebras and difference clonoids
21:20	Patrick Wynne Uniform Generation of Clonoids

Tuesday Morning

9:00	Paolo Aglianò Ideals in universal algebra invited
10:00	Mike Behrisch Axiomatising Ex-lattices
10:20	Andrew Craig Representing Sugihara monoids with binary relations
10:40	Coffee
11:10	Wesley Fussner Amalgamation in lattice-ordered groups and cancellative residuated structures invited
12:10	Dominik Lachman Effect Algebras as a Simplicially Enriched Category
12:30	Michiro Kondo On fuzzy Galois connections between L -fuzzy posets
	Lunch

Tuesday Evening

	Dinner
19:30	Zbyněk Cerman Multiplicatively transtable conic sections with respect to fixed coefficients
19:50	Adam Siwek An introduction of a new operation on a free barycentric algebra
20:10	Václav Cenker A Construction of Magmas and Related Representation
20:30	Short break
20:40	Richard Smolka T-based Orthomodular dynamic algebras
21:00	Damian Siejwa Closure operators on additively idempotent semirings

Thursday Morning

9:00	Paolo Aglianò Ideals in universal algebra invited
10:00	Thomas Vetterlein A description of Hilbert spaces as a dagger category
10:20	Sándor Radeleczki Boolean sublattices in finite partition lattices
10:40	Coffee
11:10	Wesley Fussner Amalgamation in lattice-ordered groups and cancellative residuated structures invited
12:10	Chun-Yu Lin On non-classical polyadic algebras: Soft and Hard
12:30	Helmut Länger Properties of the symmetric difference in lattices with complementation
	Lunch

Thursday Evening

	Dinner
19:30	Halimeh Moghbeli Damaneh Decompositions of Posets with least elements
19:50	Emília Halušková On algebras with easy direct limits
20:10	David Kruml Orthoset spectra of C^* -algebras
20:30	Short break
20:40	Jan Kühn Semidirect and poset product of hoops
21:00	Filip Jankovec About the lattice of sub(quasi)varieties of the class of pointed Abelian l-groups
21:20	Juanda Kelana Putra Categorical Equivalence Between Finitary Orthomodular Dynamic Algebras and Orthomodular Lattices

Friday

9:00	Paolo Aglianò Ideals in universal algebra invited
10:00	Jan Paseka Foulis quantales
10:20	Gejza Jenča Dimensionalities on monoids in Rel
10:40	Coffee
11:10	Wesley Fussner Amalgamation in lattice-ordered groups and cancellative residuated structures invited
12:10	Anna Jenčová On the combinatorial structure of types of higher order quantum maps
12:30	Ivan Chajda Orthogonality and complementation in the lattice of subspaces of a finite vector space
	Lunch

Abstracts

Invited Speakers

Ideals in universal algebra

Paolo Aglianò

invited

University of Siena

Given an algebra $\mathbf{A}$ an ideal is an “interesting subset” of the universe A , that may or may not be a subalgebra of $\mathbf{A}$; an example of the first kind is a normal subgroup of the group and of the second kind is an ideal of a commutative ring¹. Now defining what “interesting” means is largely a matter of taste; however there is a large consensus among the practitioners of the field that:

- an ideal must have a simple algebraic definition;
- ideals must be closed under arbitrary intersections, so that a closure operator can be defined in which the ideals are exactly the closed sets; this gives raise to an algebraic lattice whose elements are exactly the ideals;
- ideals must convey meaningful information on the structure of the algebra.

The three points above are all satisfied by classical ideals on lattices and of course by ideals on a set X , where we interpret a set as an algebra in which the set of fundamental operations is empty. We have however to be careful here; an ideal on a set X is an ideal (in the lattice sense) on the Boolean algebra of subsets of X . There also a significant difference between ideals on lattices and ideals on Boolean algebras; in Boolean algebras an ideal is always the 0-class of a suitable congruence of the algebra (really, of exactly one congruence), while this is not true in general for lattices. As a matter of fact, identifying the class of (lower bounded) lattices in which every ideal is the 0-class of a congruence is a difficult problem which is still unsolved, up to our knowledge.

¹we follow the modern *dictum* that every ring has a multiplicative unit...

The problem of connecting ideals of general algebras to congruence classes has been foreshadowed in [5] but really tackled by A. Ursini in his seminal paper [6]. Later, from the late 1980's to the late 1990's, A. Ursini and the author published a long series of papers on the subject ([7], [2], [3], [4], [1]); the theory developed in those papers will constitute the basis of the course.

In details first we will get acquainted with the general theory of ideals, then we will explore some particular and interesting cases in which the theory can be strengthened. Finally, if time allows, we will see some applications if the theory to specific topics in the general universal algebraic settings.

References

- [1] P. Aglianò and A. Ursini, *Ideals and other generalizations of congruence classes*, J. Aust. Math. Soc. **53** (1992), 103–115.
- [2] ———, *On subtractive varieties II: General properties*, Algebra Universalis **36** (1996), 222–259.
- [3] ———, *On subtractive varieties III: From ideals to congruences*, Algebra Universalis **37** (1997), 296–333.
- [4] ———, *On subtractive varieties IV: Definability of principal ideals*, Algebra Universalis **38** (1997), 355–389.
- [5] K. Fichtner, *Bemerkung über Mannigfaltigkeiten universeller Algebren mit Idealen*, Monatsb. Deutsch. Akad. Wiss. Berlin **12** (1970), 21–25.
- [6] A. Ursini, *Sulle varietà di algebre con una buona teoria degli ideali*, Boll. Una. Mat. Ital. **6** (1972), 90–95.
- [7] ———, *On subtractive varieties I*, Algebra Universalis **31** (1994), 204–222.

Minimal operations over permutation groups

Paolo Marimon

invited

TU Wien

We classify the possible types of minimal operations above an arbitrary permutation group. Above the trivial group, a theorem of Rosenberg says that there are five types of minimal operations. We show that above any non-trivial permutation group there are at most four such types. Indeed, except above Boolean groups acting freely on a set, there are only three. In particular, this is the case for oligomorphic permutation groups, for which we improve a result of Bodirsky and Chen. Building on these results, we answer some questions of Bodirsky related to infinite-domain constraint satisfaction problems (CSPs). This is joint work with Michael Pinsker.

Amalgamation in lattice-ordered groups and cancellative residuated structures

Wesley Fussner

invited

Czech Academy of Science

Recently, there has been tremendous progress in developing a systematic theory of amalgamation in residuated structures, but extending this progress to cancellative residuated structures has proven to be one of the most significant challenges in this body of work. This series of lectures focuses on this problem, in particular charting the difficult terrain around amalgamation in lattice-ordered groups. We discuss both the landmark results in this field as well as new avenues toward the resolution of long open problems.

Monday

How generalized quasiorders appear in rectangular algebras

Reinhard Pöschel

TU Dresden

Rectangular algebras generalize rectangular bands (and include Płonka's diagonal algebras). Generalized quasiorders are a generalization of (binary) quasiorders to higher-arity relations and describe relations $\rho \subseteq A^m$ with the property that an operation $f : A^n \rightarrow A$ preserves ρ if and only if all unary translations of f do it. It turns out that, e.g., the graphs of fundamental operations of a rectangular algebra are generalized quasiorders, more precisely generalized partial orders. This is a joint talk with D. Jakubíková-Studenovská and S. Radeleczki.

Role of weak congruences in theoretical and practical applications

Andreja Tepavčević

SANU & University of Novi Sad

Weak congruences on algebras were introduced at the end of the last century as relations that satisfied all features of congruences except reflexivity. All weak congruences on an algebra form an algebraic lattice, which has been used for the investigation of structural properties of algebras. In this talk, we will mention two aspects of applying weak congruences. The first is connected to the representation of various classes of groups and group-like algebras. Namely, we characterized several classes of groups by their weak congruence lattices, for example, abelian groups, Hamiltonian groups, nilpotent groups, solvable groups, etc. Another aspect is connected to applications with so-called Ω -algebras, which are ordinary algebras with generalized equality (a particular mapping from $A \times A$ to a complete lattice). Particular Ω -algebras like Ω -vector spaces have a big role in the approximate solving of systems of relational equations. Both aspects will be briefly presented with some examples.

This research was supported by the Science Fund of the Republic of Serbia, # Grant no 6565, Advanced Techniques of Mathematical Aggregation and Approximative Equations Solving in Digital Operational Research-AT-MATADOR

A lope towards loops: temporal digraphs

Johanna Brunar

TU Wien

The Bodirsky-Kára classification of all tractable temporal constraint languages stands as one of the earliest and most seminal complexity classifications within infinite-domain Constraint Satisfaction Problems (CSPs). We revisit this classification and show that tractable temporal languages have limited expressive power as measured by the graphs they can pp-interpret. This limitation leads to many hitherto unknown algebraic consequences. In particular, we confirm that for temporal constraint languages, the existence of a 4-ary pseudo-Siggers polymorphism is equivalent to the existence of a 6-ary one. While for finite-domain CSPs, the presence of the non-pseudo version of either of these polymorphisms is known to equivalently characterise tractability, the existence of a 6-ary pseudo-Siggers polymorphism has been conjectured, within the Bodirsky-Pinsker conjecture, to separate the hard from the tractable CSPs.

Kőnig = Ramsey

Maximilian Hadek

Charles University

Ramsey's theorem and Kőnig's tree lemma are two famous results from infinite combinatorics, which at first glance seem unrelated. The former talks about colouring subsets of finite sets, while the latter is more akin to a choice principle, allowing us to find infinite paths within trees. They share one moral similarity though: both establish order within seemingly chaotic, infinite objects. We make this precise, by proving that a generalized Kőnig's lemma holds on a category if and only if it has the Ramsey property.

Algebraicity is irrelevant (in the dichotomy conjecture for infinite-domain constraint satisfaction problems)

Moritz Albert Schöbi

TU Wien

The Bodirsky-Pinsker conjecture is an infinite counterpart to the Feder-Vardi dichotomy conjecture for Constraint Satisfaction Problems (CSPs) with finite templates. While the latter has been confirmed independently by Bulatov and Zhuk, the former remains wide open. In this talk, we shed light on two meta-problems regarding the scope of this conjecture. Our first result provides a significant structural simplification: we prove that the conjecture is equivalent to its restriction to templates without algebraicity, a crucial assumption in many powerful classification methods. The second result provides a simplification of algebraic nature: any algebraic condition characterizing any complexity class within the conjecture must be satisfiable by injections. In particular, this offers insight into which universal-algebraic conditions for the complexity of finite-template CSPs may be successfully lifted to the infinite case. This is joint work with Michael Pinsker, Jakub Rýdval and Christoph Spiess.

An ω -categorical cheese platter for finite-domain Promise Constraint Satisfaction Problems

Christoph Spiess

TU Wien

The Bodirsky-Pinsker dichotomy conjecture is of central importance in the field of infinite-domain Constraint Satisfaction Problems (CSPs). It extends the finite-domain complexity dichotomy confirmed by Bulatov and Zhuk to a certain class of well-behaved infinite templates.

We demonstrate that every non-trivially tractable infinite template within the scope of the Bodirsky-Pinsker conjecture can be transformed into a witness for the tractability of a finite-domain Promise Constraint Satisfaction Problem (PCSP) that is not finitely tractable. Specifically, we show that, up to a Datalog-reduction, it serves as such a witness via the so-called sandwich method. This result strengthens a novel connection between the Bodirsky-Pinsker conjecture and finite-domain PCSPs.

Clones between Group $(\mathbb{Z}_8, +)$ and Ring $(\mathbb{Z}_8, +, \cdot)$

Radka Schwartzová, Miroslav Ploščica

Pavol Jozef Šafárik University in Košice

A clone on a set is a family of operations that includes all projections and is closed under compositions. An important example is the clone of polynomial operations on groups or rings.

Let p be a prime number. Our work continues the line of research initiated by Rosenberg (1970), Krokhin et al. (1997), Idziak and Bulatov (2003), which investigates the interval between the clone of group polynomials and the clone of ring polynomials on the same set $\mathbb{Z}_p$, respectively $\mathbb{Z}_{p^2}$, within the clone lattice on the corresponding set. The talk will focus on a complete description of all clones generated by polynomials with even coefficients, except for the univariate polynomials, that lie between the clone of group polynomials and the clone of ring polynomials on the set $\mathbb{Z}_{p^3}$, for $p = 2$.

Meet-irreducibility of congruence lattices of prime-cycled algebras

Juraj Hirjak, Lucia Kőszegyová

Pavol Jozef Šafárik University in Košice

The set of all congruences on an algebra (A, F) forms a lattice $\text{Con}(A, F)$. Similarly, the set of all congruence lattices of all algebras defined on a fixed base set A forms a lattice. The aim of this talk is to explore some of the meet-irreducible elements of the lattice of congruence lattices of all algebras defined on a finite set A . It is known that all these meet-irreducible elements are congruence lattices of monounary algebras, which can be visualized as planar graphs. Using this visualization, we study the meet-irreducible elements, specifically in the case when each cycle of (A, F) contains a prime number of elements, i.e. we focus on prime-cycled algebras.

Mal'cev algebras and difference clonoids

Michael Kompatscher

Charles University

While Mal'cev algebras are often considered to be among the most tame structures in universal algebra, surprisingly many basic questions about them still remain open: When ordered by term equivalence, can there be an infinite anti-chain of Mal'cev algebras on a finite set? Which Mal'cev algebras have finite equational bases? Is the subpower membership problem for finite Mal'cev algebras always solvable in polynomial time? In this talk, I would like to introduce the difference clonoid as a tool that can be useful to discuss these questions, in particular, in the setting of nilpotent algebras.

Uniform Generation of Clonoids

Patrick Wynne

Charles University

A clonoid is a set of functions from finite powers of an algebra A into an algebra B that is closed with respect to precomposition with term functions of A as well as with postcomposition with term functions of B . Clonoids between modules (or more generally, abelian Mal'cev algebras) have become a useful tool in the study of nilpotent Mal'cev algebras. We introduce the concept of uniformly generated clonoids. We then show that if A is a finite distributive module and B is a module of coprime order then every clonoid from A to B is uniformly generated and the number of clonoids is finite.

Tuesday

Axiomatising Ex-lattices

Mike Behrisch

TU Wien

In the 2023 paper ‘A fundamental non-classical logic’, W. Holliday introduced from a proof-theoretic perspective a *fundamental logic*, which is a common generalisation of both orthologic (a weakening of quantum logic) and intuitionistic logic. In fact, Holliday proved that his set of Fitch-style natural deduction rules leads to *the* weakest introduction-elimination logic. Algebraically, fundamental logic corresponds to bounded lattices $\mathbf{L}$ with a *weak pseudocomplementation* $\bar{}$, i.e., the pair $(\bar{}, \bar{})$ forms a Galois connection on $\mathbf{L}$ and $\mathbf{L} \models x \cdot \bar{x} \approx 0$, where multiplication denotes conjunction (meet).

Such structures are named *fundamental lattices* by Aguilera and Massas who recently gave an answer to the problem posed by Holliday of describing the intersection of orthologic and intuitionistic logic. They extend the axiomatisation of fundamental lattices by the complicated axiom (Ex)

$$\overline{x(yw + yu)} \cdot x(w + v) \cdot \bar{z} \leq \bar{xz} \cdot (xw + xv + z) \cdot (y(w + u) + \overline{y(w + u)})$$

with universally quantified variables, where $+$ denotes disjunction, juxtaposition denotes multiplication (conjunction), binding stronger than disjunction, and the vinculum represents negation. Aguilera and Massas name the subclass of fundamental lattices satisfying the axiom (Ex) *Ex-lattices* and prove that this class of structures can also be axiomatised by the axioms of fundamental lattices and three laws using at most four variables. They ask, however, about the three-variable fragment of Ex-logic.

We will show that Ex-lattices form a variety that can be axiomatised by the bounded lattice equations and seven additional identities (or universal inequalities) each mentioning at most three variables. All but one of these seven are common identities of ortholattices and pseudocomplemented lattices, that is, we can pinpoint the influence of distributivity coming from the theory of Heyting lattices, i.e., from intuitionistic logic.

This research was funded in part by the Austrian Science Fund (FWF) 10.55776/PIN5424624.

Representing Sugihara monoids with binary relations

Andrew Craig

University of Johannesburg

Sugihara monoids can be defined as commutative idempotent distributive residuated lattices with an order-reversing involution. These algebras are used as algebraic semantics for certain relevance logics. We will consider Sugihara monoids as algebras in the more general class of distributive involutive FL-algebras (DInFL-algebras). Our first result is to extend the so-called generalised ordinal sum construction (Galatos 2005) from residuated lattices to DInFL-algebras. Then, combining our generalised ordinal sum construction with a method for representing DInFL-algebras, we prove that every finite Sugihara chain is representable as an algebra of binary relations. Lastly, we use ultraproducts to show that every Sugihara monoid is representable in this way. (Joint work with Wilmari Morton and Claudette Robinson.)

Effect Algebras as a Simplicially Enriched Category

Dominik Lachman

Palacký University Olomouc

Effect algebras are certain partial monoids that arise naturally in the study of quantum logic. Our work is based on two key observations. First, every effect algebra can be viewed as a Frobenius algebra internal to the category of sets and relations. Second, the category of such Frobenius algebras embeds fully faithfully into a certain category of simplicial sets. Using these observations, we organize effect algebras into a simplicially enriched category, denoted $\mathbf{EA}$. We investigate properties of a mapping space $\text{Map}(E, F)$, for $E, F \in \mathbf{EA}$. Additionally, we consider the non-commutative generalization to pseudo effect algebras, $\mathbf{PEA}$, which admits a well-behaved concept of conjugation. We show that several properties of conjugation have a natural interpretation from the combinatorial perspective of simplicial sets.

On fuzzy Galois connections between L -fuzzy posets

Michiro Kondo

Tokyo Denki University

We consider properties of fuzzy Galois connections between L -fuzzy posets for complete residuated lattices L and show their properties by operator-based algebraic methods. We also give a simple condition for the existence of right adjoint map.

For given two partially ordered sets (simply called posets) $(X, \leq_X)$ and $(Y, \leq_Y)$, these sets have some relationship between them when there are maps such as order-preserving maps, injective maps. Especially, a pair (f, g) of maps, called a pair of Galois connection, is very important notion for posets. Roughly speaking, this indicates that if there is a Galois connection between posets, then a certain property in one poset is "similarly" reflected in another poset and vice versa. Therefore, Galois connections arise very frequently in many fields on mathematics, computer science and so on, and their results make us the situation much more easily understood.

Let $(X, \leq_X)$, $(Y, \leq_Y)$ be posets and $f : X \rightarrow Y$, $g : Y \rightarrow X$ maps. A pair (f, g) of maps is called a Galois connection if $f(x) \leq_Y y$ if and only if $x \leq_X g(y)$ for all $x \in X, y \in Y$. It is easy to show that (f, g) is a Galois connection if and only if both f, g are order-preserving and $x \leq_X gf(x)$ and $fg(y) \leq_Y y$ for all $x \in X, y \in Y$. In this case, two maps f, g are called the left and the right adjoint, respectively.

We have a familiar example of Galois connection in the classical set theory. Let X, Y be non-empty sets and $f : X \rightarrow Y$ a map. It is obvious that both $(2^X, \subseteq)$ and $(2^Y, \subseteq)$ are posets. Then, a pair (f, f^{-1}) of maps is a Galois connection between them, because $f(A) \subseteq B$ if and only if $A \subseteq f^{-1}(B)$ for all $A \in 2^X, B \in 2^Y$.

The notion of Galois connections are generalized to the case of L -fuzzy sets for complete residuated lattices L by Bělohlávek (1999), where L^X is used instead of 2^X . A pair (f, g) of maps is called a *fuzzy Galois connection* between L -fuzzy posets (L^X, Sub_X) and (L^Y, Sub_Y) for maps $f : L^X \rightarrow L^Y$ and $g : L^Y \rightarrow L^X$, if f, g satisfies the conditions: For all $A, A_1, A_2 \in L^X$, $B, B_1, B_2 \in L^Y$,

$$\begin{aligned} \text{(BFG1)} \quad & \text{Sub}_X(A_1, A_2) \leq \text{Sub}_Y(f(A_1), f(A_2)) \text{ and} \\ & \text{Sub}_Y(B_1, B_2) \leq \text{Sub}_X(g(B_1), g(B_2)); \end{aligned}$$

$$\text{(BFG2)} \quad A \leq gf(A), \quad B \leq fg(B),$$

where Sub_X is defined by

$$\text{Sub}_X(A_1, A_2) = \bigwedge_{x \in X} (A_1(x) \rightarrow A_2(x)).$$

Sub_Y is similarly defined.

It was also proved that fuzzy Galois connections are in one-to-one correspondence with binary fuzzy relations.

Since a map $\text{Sub}_X : L^X \times L^X \rightarrow L$ satisfies the following conditions

$$(\text{Sub1}) \text{Sub}_X(A, A) = 1 \quad (\forall A \in L^X);$$

$$(\text{Sub2}) \text{Sub}_X(A_1, A_2) = \text{Sub}_X(A_2, A_1) = 1 \text{ implies } A_1 = A_2 \text{ for all } A_1, A_2 \in L^X;$$

$$(\text{Sub3}) \text{Sub}_X(A_1, A_2) \odot \text{Sub}_X(A_2, A_3) \leq \text{Sub}_X(A_1, A_3) \text{ for all } A_1, A_2, A_3 \in L^X,$$

it can be considered as a fuzzy partial order on the L -fuzzy sets. Yao and Lu (2009) considered the notion L -fuzzy posets as the generalization of posets and proved some fundamental properties about L -fuzzy posets and fuzzy Galois connections.

In this talk, we give operator-based proofs to their results, which make such results can be extended easily to more general cases. Moreover, we show that a new Galois connection arises between L -fuzzy posets.

Multiplicatively transtable conic sections with respect to fixed coefficients

Zbyněk Cerman

Tomas Bata University in Zlín

The geometric mean possesses several significant properties. One of these properties is the preservation of the result when multiplying and dividing by the same value. Consequently, if one input value is increased and another is decreased, the result remains unchanged. This property is referred to as “transtability”. The implications of transtability are observable in numerous mathematical theories, although its most prevalent application is in the aforementioned aggregation functions. In the early stages of this theory’s exploration, the concept of so-called additive transtability was explored. This concept is associated with the arithmetic mean, where the same value was added and subtracted to different input values. Furthermore, the correlation between additive transtability and conic sections is immediately evident. This is attributable to the preservation of coefficients, which results in a multitude of intriguing properties. The objective of this paper is to point out the significance of the second form of transtability, termed so-called multiplicative, which involves multiplying and dividing by the same number. There are a number of complications that are associated with multiplicative transtability and conic sections. Our aim is to describe these complications and resolve them as efficiently as possible. We will focus on two primary issues that can unify transtable conic sections. The first issue describes their intersections and the second issue deals with the set of centers. We demonstrate that common intersections for multiplicative transtable conic sections exist solely under specific assumptions, such as the elimination of a particular coefficient.

An introduction of a new operation on a free barycentric algebra

Adam Siwek

Warsaw University of Technology

Barycentric algebras, originating from Möbius's use of barycentric coordinates, were introduced in the 1940s as an axiomatization of real convex sets, presented algebraically via binary operations. They unify the concepts of convexity (every convex set is a barycentric algebra) and order (the variety of semilattices is a subvariety of the variety of barycentric algebras). The free barycentric algebra over a finite set X can be interpreted as the set of all probability distributions of random variables on X . Moreover, the free barycentric algebra over a finite set $X \cup \{\perp\}$ corresponds to the set of all subprobability distributions of random variables on X . Our aim is to study the structures we have built based on free barycentric algebras by introducing a new operation. Initially, we focused on algebraic properties, sets of isomorphisms, and order relations within this new structure. However, the idea emerged to explore, more specifically, the space of probability functions over a group. The results we obtained reveal surprising applications and interpretations, particularly concerning the entropy (in the sense of information theory) of random distributions over groups.

A Construction of Magmas and Related Representation

Václav Cenker

Palacký University Olomouc

In this talk, we present the following construction. Given a magma $\mathbf{G} = (G, \cdot)$ and a biunary algebra $\mathbf{I} = (I, \lambda, \rho)$ (an algebra with two unary operations), we define a new magma $\mathbf{G}^{\mathbf{I}} = (G^I, \cdot)$ by $(x \cdot y)(i) = x(\lambda i) \cdot y(\rho i)$ for any $x, y \in G^I$ and any $i \in I$. Our main goal is to characterize the variety generated by all magmas $\mathbf{G}^{\mathbf{I}}$ obtained in this way, where $\mathbf{G}$ and $\mathbf{I}$ are taken from arbitrary (but fixed) varieties of magmas and biunary algebras, respectively.

T-based Orthomodular dynamic algebras

Richard Smolka

Masaryk University

We extend the foundational work of Kishida, Rafiee Rad, Sack, and Zhong on orthomodular dynamic algebras (ODAs), who constructed dynamic structures by closing the set of Sasaki projections on a complete orthomodular lattice under composition. In contrast, we begin with an arbitrary set G of linear operators, along with certain other modifications, on the lattice L , required only to include all Sasaki projections (plus possibly other morphisms), and freely generate an involutive quantale $\mathbf{FQ}(G)$ over this set. This generalization preserves the orthomodular character of the dynamics while permitting a more expressive range of quantum actions. We expect to recover analogous equivalence and representation results for this enriched setting.

Closure operators on additively idempotent semirings

Damian Siejwa

Warsaw University of Technology

The talk will focus on various closure operators defined on semigroups and additively idempotent semirings. In particular, we will be interested in construction of spectral spaces. During the talk, I will present several examples of closure operators and spectral spaces, and I will discuss how closure operators can be used to prove the algebraicity of certain lattices.

Thursday

A description of Hilbert spaces as a dagger category

Thomas Vetterlein

Johannes Kepler University Linz

Complex Hilbert spaces provide the basic mathematical model used in quantum physics. Why this choice is suitable has been a topic of extensive research. In particular, the question has been raised whether the model can be reduced to a simpler one. Indeed, Hilbert spaces can, for instance, be characterised as certain algebraic or relational structures. Moreover, it has recently turned out to be possible to describe Hilbert spaces by purely categorical means.

We elaborate on the latter possibility. We work in the framework of dagger categories; a dagger is an involutory contravariant endofunctor that is the identity on objects. The complex Hilbert spaces and bounded linear maps between them form a category $\mathcal{Hil}$ and the usual adjoint makes $\mathcal{Hil}$ into a dagger category. We present a list of categorical axioms characterising $\mathcal{Hil}$.

Related results are due to Chris Heunen and Andre Kornell (PNAS 2021), who work, however, in the framework of monoidal categories. A further approach, which characterises the Hilbert spaces over one of the three classical division rings, is due to Stephen Lack and Shay Tobin (Appl. Categor. Struct. 2025).

The work is done jointly with Jan Paseka.

This research was funded in part by the Austrian Science Fund (FWF) 10.55776/ PIN5424624 and the Czech Science Foundation (GAČR) 25-20013L.

Boolean sublattices in finite partition lattices

Sándor Radeleczki

University of Miskolc

Let $\text{Part}(U)$ denote the partition lattice of a finite universe U . We show that any highest dimensional Boolean sublattice of $\text{Part}(U)$ can be formed using all partitions whose blocks are some subtrees of a tree with vertex set U . We prove that a maximal Boolean sublattice of $\text{Part}(U)$ always contains the least $\triangle$ and the largest ∇ elements of $\text{Part}(U)$. We characterize the maximal Boolean sublattices of $\text{Part}(U)$ by a certain condition imposed on the circles of a linear hypergraph induced by the atoms of a Boolean sublattice $\mathcal{B} \leq \text{Part}(U)$ with $\triangle, \nabla \in \mathcal{B}$ on the set U .

On non-classical polyadic algebras: Soft and Hard

Chun-Yu Lin

Czech Academy of Science

Polyadic algebras, introduced by Paul R. Halmos, provide an algebraic framework for classical first-order logic. Building on generalizations via relation algebras by H. Andréka, I. Németi, I. Sain, and G. Sági, as well as polyadic VB-algebras studied by D. Pigozzi and A. Salibra, various fragments of first-order logic—including Rasiowa’s implicative predicate logic—can be algebraically characterized using polyadic algebras. In this talk, we first define polyadic VB-algebras based on algebraically implicative logics. We then establish a functional representation theorem for these polyadic algebras. Finally, we demonstrate how this theorem reveals the connection between the VB-calculus and predicate algebraically implicative logic, thereby linking the “soft” and “hard” approaches to algebraizing first-order logic as advocated by J. M. Font.

Properties of the symmetric difference in lattices with complementation

Helmut Länger

TU Wien & Palacký University Olomouc

The symmetric difference in Boolean lattices can be defined in two different but equivalent forms. However, it can be introduced also in every bounded lattice with complementation where these two forms need not coincide. We study lattices with complementation where these two expressions coincide. It is well-known that the symmetric difference is associative in every Boolean lattice. We prove that this is just the property of Boolean lattices, namely the symmetric difference in a lattice with complementation is associative if and only if this lattice is Boolean. Similarly, we prove that a lattice with complementation is Boolean if and only if the symmetric difference satisfies a certain simple identity in two variables.

Decompositions of Posets with least elements

Halimeh Moghbeli Damaneh¹, Konrad Pióro²

¹FAU Erlangen-Nürnberg, ²University of Warsaw

We show that direct product decompositions (into two factors) of a poset $(P, \leq, \perp)$ with a least element $\perp$ bijectively correspond to complement neutral elements A of the lattice of finitely stable subposets of P (where a subposet is finitely stable if it is an order ideal closed under all existing finite suprema) such that A and its complement B satisfy the following two conditions: (i) for each $p \in P$, sets $\{a \in A: a \leq p\}$ and $\{b \in B: b \leq p\}$ have greatest elements; (ii) a supremum of a and b exists in P for all $a \in A$ and $b \in B$. The same result holds also for the lattice of stable subposets of P , where a subposet is stable if it is an order ideal closed under all existing suprema. These results generalize the well-known analogous result from lattice theory. We also prove that if we take the lattice of stable subposets of P and omit (ii), then each element of P has a unique representation as a supremum of two elements, one from A and the other from B . This kind of decompositions is introduced in Scott-domain theory and our result is a generalization of an analogous theorem for Scott-domains. Moreover, we describe what kinds of decompositions of a poset P with a least element correspond to complement neutral elements of lattices of finitely stable and stable subposets of P if we assume no additional conditions for these elements.

On algebras with easy direct limits

Emília Halušková

Slovak Academy of Sciences

The direct limit construction belongs to basic tools used in Universal algebra. We will focus on direct limit families in which only one algebra occurs.

Let $\mathcal{A}$ be an algebra. We denote by $\mathbf{LA}$ the class of all isomorphic copies of direct limits which can be obtained from $\mathcal{A}$ and we denote by $\mathbf{RA}$ the set of all retracts of $\mathcal{A}$. Then $\mathbf{RA} \subseteq \mathbf{LA}$. We will say that $\mathcal{A}$ *has easy direct limits* if every algebra from $\mathbf{LA}$ is isomorphic to a retract of $\mathcal{A}$.

Finite algebras are with easy direct limits. We will present that:

- Vector spaces with easy direct limits are exactly finite dimensional ones.
- There is a simple algebra such that it has no easy direct limits.
- The additive group of rational numbers has easy direct limit and additive groups of integers and real numbers do not.

This is the joint work with Małgorzata Jastrzębska.

Orthoset spectra of C^* -algebras

David Kruml

Masaryk University

Orthoset spectrum of a C^* -algebra is a lattice of q -open sets (isomorphic to closed right ideals) with the orthogonality relation on its maximal elements. The spectrum could be a promising invariant of the C^* -algebra. For now, we can reconstruct the C^* -algebra from its orthoset spectrum unless it has a non-trivial 2-homogeneous quotient. However, in such a case it seems that the spectrum still carries the needed information.

Semidirect and poset product of hoops

Jan Kühr

Palacký University Olomouc

We will compare two types of semidirect product of hoops and their residuation subreducts with the so-called poset product.

About the lattice of sub(quasi)varieties of the class of pointed Abelian l-groups

Filip Jankovec

Charles University

The varieties of MV-algebras, classified by Komori correspond to varieties of positively pointed abelian l-groups, as shown by Young, via the Mundici functor. We will generalize this result to classification of all subvarieties of pointed Abelian l-groups. We will moreover generalize this result to the class of quasivarieties generated by chans, using the classification of universal theories of MV-algebras provided by Gispert.

Categorical Equivalence Between Finitary Orthomodular Dynamic Algebras and Orthomodular Lattices

Juanda Kelana Putra

Masaryk University

This talk reveals a categorical equivalence connecting two distinct quantum logic structures. The first is the orthomodular lattice, an algebraic system designed to formalize the properties of quantum systems. The second is a finitary orthomodular dynamic algebra, a specialized development of the orthomodular dynamic algebra where the underlying quantum actions are restricted to be finitary. The applicability of the result extends to more specialized lattices, such as Hilbert lattices of closed subspaces of a Hilbert space, beyond general orthomodular lattices. As these lattice structures exhibit connections to a diverse array of quantum structures, the established equivalence categorically bridges unital involutive m-semilattices with a broad spectrum of quantum formalisms.

Friday

Foulis quantales

Jan Paseka

Masaryk University

We have established a novel correspondence between complete orthomodular lattices and a particular class of quantales. In the forward direction, given any complete orthomodular lattice X , we construct an associated Foulis quantale $Lin(X)$ consisting of join-preserving mappings on X . This construction enables us to view X as a left $Lin(X)$ -module, thereby providing a new fuzzy-theoretic perspective on complete orthomodular lattices while introducing an exterior implication operation on these structures. In the reverse direction, we show that every Foulis quantale Q gives rise to a complete orthomodular lattice $[Q]$ that naturally carries the structure of a left Q -module. Furthermore, this construction yields a canonical Foulis quantale homomorphism from Q to $Lin([Q])$. This bidirectional relationship creates a bridge between the algebraic theory of orthomodular lattices and the categorical framework of quantales, potentially opening new avenues for investigating quantum logical structures through module-theoretic methods.

Dimensionalities on monoids in **Rel**

Gejza Jenča

Slovak University of Technology

Motivated by the fact that a dimension relation on an orthomodular lattice can be considered as a particular internal monad on a (partial) monoid in the 2-category of sets and relations, we examine the following type of equivalences on a monoid $(A, ., e)$, which we call *dimensionality*:

- If $a.b \sim c$, there exist $a' \sim a$ and $b' \sim b$ such that $a'.b' = c$.
- If $e \sim x$, then $e = x$.

We show that there are many examples of dimensionality arising in mathematics: on groups, modular lattices, orthomodular lattices, etc. In particular, orbits of a group acting on a monoid always give rise to a dimensionality. If the monoid is a group, the quotient monoid is a hypergroup.

We prove that if $\sim$ is a dimensionality on a monoid A , then $A/\sim$ is a *relational monoid*, that means, a monoid in the monoidal category of sets and relation **Rel**. This motivates us to extend the notion of dimensionality to all monoids in **Rel**.

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On the combinatorial structure of types of higher order quantum maps

Anna Jenčová

Slovak Academy of Sciences

Higher order quantum maps (HOM) form a hierarchy describing most general protocols of quantum information processing. This hierarchy that can be built recursively, starting from elementary systems and including at each step all admissible transformations between objects constructed in previous steps. Loosely speaking, the HOM can be described as "transformations between transformations" satisfying certain admissibility conditions.

The recursive construction leads to a formalism of types of HOMs (Bisio and Perinotti, 2019), which have a combinatorial characterization that can be given in terms of certain boolean functions - the type functions. We show that to every type function we can find a finite poset which fully characterizes the type, and that the causal (or signaling) structure of HOMs of the given type can be inferred from this poset.

The purpose of this talk is to introduce these posets and the related problems to the algebraic community.

Orthogonality and complementation in the lattice of subspaces of a finite vector space

Ivan Chajda

Palacký University Olomouc

We investigate the lattice $L(V)$ of subspaces of an m -dimensional vector space V over a finite field $GF(q)$ with a prime power $q = p^n$ together with the unary operation of orthogonality. It is well-known that this lattice is modular and that the orthogonality is an antitone involution. The lattice $L(V)$ satisfies the chain condition and we determine the number of covers of its elements, especially the number of its atoms. We characterize when orthogonality is a complementation and hence when $L(V)$ is orthomodular. For q being a prime number and $m = 2$ we characterize orthomodularity of $L(V)$ by a simple condition.

Participants

Name	Affiliation
Paolo Aglianò	University of Siena
Tamás Ágoston	University of Szeged
Alexey Barsukov	Charles University
Libor Barto	Charles University
Mike Behrisch	TU Wien
Bernardo Bocci	University of Siena
Jan Bok	Charles University
Johanna Brunar	TU Wien
Václav Cenker	Palacký University Olomouc
Zbyněk Cerman	Tomas Bata University in Zlín
Ivan Chajda	Palacký University Olomouc
Andrew Craig	University of Johannesburg
Petr Emanovský	Palacký University Olomouc
Roman Feller	TU Wien
Wesley Fussner	Czech Academy of Science
Maximilian Hadek	Charles University
Radomír Halaš	Palacký University Olomouc
Emília Halušková	Slovak Academy of Sciences
Miroslav Haviar	Matej Bel University in Banská Bystrica
Juraj Hirjak	Pavol Jozef Šafárik University in Košice
Filip Jankovec	Charles University
Přemysl Jedlička	Czech University of Life Sciences Prague
Gejza Jenča	Slovak University of Technology
Anna Jenčová	Slovak Academy of Sciences
Kamilla Kátaí-Urbán	University of Szeged
Piotr Kosela	Maria Curie-Skłodowska University
Miroslav Kolařík	Palacký University Olomouc
Michael Kompatscher	Charles University
Michiro Kondo	Tokyo Denki University
Lucia Kőszegyová	Pavol Jozef Šafárik University in Košice
David Kruml	Masaryk University
Jan Kühn	Palacký University Olomouc
Dominik Lachman	Palacký University Olomouc

Name	Affiliation
Helmut Länger	TU Wien & Palacký University Olomouc
Chun Yu Lin	Czech Academy of Science
Paolo Marimon	TU Wien
Halimeh Moghbeli Damaneh	FAU Erlangen-Nürnberg
Jan Paseka	Masaryk University
Lukas Pieper	Charles University
Jozef Pócs	Slovak Academy of Sciences
Reinhard Pöschel	TU Dresden
Juanda Kelana Putra	Masaryk University
Sándor Radeleczki	University of Miskolc
Moritz Albert Schöbi	TU Wien
Radka Schwartzová	Pavol Jozef Šafárik University in Košice
Damian Siejwa	Warsaw University of Technology
Adam Siwek	Warsaw University of Technology
Richard Smolka	Masaryk University
Christoph Spiess	TU Wien
David Stanovský	Charles University
Miloslav Štěpán	Johannes Kepler University Linz
Csaba Szabó	Eötvös Loránd University
Andreja Tepavčević	SANU & University of Novi Sad
Kristóf Varga	University of Szeged
Thomas Vetterlein	Johannes Kepler University Linz
Paul Winkler	TU Dresden
Patrick Wynne	Charles University
Dmitriy Zhuk	Charles University

MondayTuesday

9:00	Paolo Agliano	Paolo Agliano
10:00	Reinhard Pöschel	Mike Behnisch
10:20	Andreja Tepavčević	Andrew Craig
10:40	<i>Coffee</i>	
11:10	Paolo Marimon	Wesley Fussner
12:10	Johanna Brumar	Dominik Lachman
12:30	Maximilian Hadek	Michiro Kondo
	<i>Lunch</i>	

ThursdayFriday

	Paolo Agliano	Paolo Agliano
	Thomas Vetterlein	Jan Paseka
	Sándor Radeleczki	Gejza Jenča
	<i>Coffee</i>	
	Wesley Fussner	Wesley Fussner
	Chun Yu Lin	Anna Jenčová
	Helmut Längler	Ivan Chajda
	<i>Lunch</i>	

Wednesday excursion

	<i>Dinner</i>		<i>Dinner</i>
19:30	Moritz Albert Schöbi	Zbyněk Cerman	Halimeh Moghbeli Damaneh
19:50	Christoph Spiess	Adam Siwek	Emília Halušková
20:10	Radka Schwartzová	Václav Cenker	David Kruml
20:30	<i>Short break</i>		<i>Short break</i>
20:40	Juraj Hirjak	Richard Smolka	Jan Kühr
21:00	Michael Kompatscher	Daniam Siejwa	Filip Jankovec
21:20	Patrick Wymne		Juanda Kelana Putra