Lattices isomorphic to subsemilattice lattices of finite trees

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$\langle S, \wedge \rangle$ is a *semilattice*, if $\wedge$ is associative, commutative, and idempotent.

Sub(S) denotes the subsemilattice lattice.

Theorem 1 [*Repnitskiĭ, 1996*]
Any lattice embeds into a subsemilattice lattice.

Theorem 2 [*Repnitskiĭ, 1993; Adaricheva, 1996*]
L *embeds into a finite subsemilattice lattice iff* L *is finite lower bounded (in the sense of McKenzie).*

A connected poset $\langle P, \leq \rangle$ is a *tree* if $\downarrow p$ is a chain for any $p \in P$.

$\mathcal{T}$ denotes the class of semilattices which are trees.

Theorem 3 *The class of lattices which embed into subsemilattice lattices of finite trees is axiomatized by identities within the class of finite lattices.*

Theorem 4 $L \cong \mathsf{Sub}(T)$ *for a finite tree* T *iff* L *is finite atomistic and* $L \models$ (T_2), (T), (M), (Tr).

(T_2), (T), (M) are identities;

(Tr) is a first-order sentence saying that there is only one D-minimal element which is not join-prime.

Corollary 5 *The class of lattices isomorphic to subsemilattice lattices of finite trees is finitely axiomatizable within the class of finite lattices.*